Is torsion needed in theory of gravity?

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(February 3, 2008)

Abstract

It is known that General Relativity (**GR**) uses Lorentzian Manifold $(M_4; g)$ as a geometrical model of the physical space–time. M_4 means here a four–dimensional differentiable manifold endowed with Lorentzian metric g. The metric g satisfies Einstein equations. Since the 1970s many authors have tried to generalize this geometrical model of the physical space–time by introducing torsion and even more general metric–affine geometry. In this paper we discuss status of torsion in the theory of gravity. At first, we emphasize that up to now we have no experimental evidence for the existence of torsion in Nature. Contrary, the all experiments performed in weak gravitational field (Solar System) or in strong regime (binary pulsars) and tests of the Einstein Equivalence Principle (**EEP**) confirmed **GR** and Lorentzian manifold $(M_4; g)$ as correct geometrical model of the physical space–time. Then, we give theoretical arguments against introducing of torsion into geometrical model of the physical space–time. At last, we conclude that the general–relativistic model of the physical space–time is sufficient and it seems to be the most satisfactory.

I. INTRODUCTION

In the 70ths of the XX Century many research workers have introduced torsion into theory of gravity[1][1,2,3]. The motivation (only theoretical) was the following:

1. Analogies **GR** with the continual theory of dislocations (Theory of generalized Cosserat continuum) led to heuristic arguments for a metric space–time with torsion, i.e., to Riemann–Cartan space–time.

2. Study of spinning matter in **GR** led some people to conclusion that the canonical energy–momentum tensor of matter $_cT_i{}^k$ is the source of the curvature and the canonical intrinsic spin density tensor $_cS^{ikl} = (-)_cS^{kil}$ is the source of torsion of the underlying space–time. From these studies Einstein–Cartan–Sciama–Kibble (**ECSK**) theory was originated and its generalizations.

3. Trials to formulate theory of gravity as a gauge theory for Lorentz group L or for Poincare' group P led to a space–time endowed with a metric– compatible connection which could have (but not necessarily) non–vanishing torsion, i.e., led to the Riemann–Cartan space–time [4–6].

The above motivation *is not convincing*. For example, the often used argument for torsion (It followed from 2.) based on (non-homogeneous) holonomy theorem [1][2] holds only if one uses *Cartan displacements* [7]. Ordinary parallell displacements gives only Lorentz rotations (= homogeneous holonomy group) even in a Riemann-Cartan space-time [7]. Moreover, there are other geometrical interpretation of torsion, e.g., Bompiani [8] *connects torsion with rotations in tangent spaces*, not with translations. We also needn't generalize **GR** in

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[1]We omit here older trials to introduce torsion because they have only historical meaning.

[2]This theorem says that torsion gives translations and curvature gives Lorentz rotations in tangent spaces of a Riemann-Cartan manifold during displacements along loops.

order to get a gauge theory with L or P as a gauge group [9]. Notice especially that in the *Asthekar's variables* [10,11] the ordinary **GR** with Levi–Civita connection *itself gives* an example of the simplest model of a gauge theory of gravity.

Up to now, *we have no experimental evidence for existence of torsion in Nature.* Contrary, the all experiments confirmed with a very high precision ($\sim 10^{-14}$) Einstein Equivalence Principle (**EEP**) and (with a smaller precision) General Relativity (**GR**) equations [12–15]. Here by **EEP** we mean a formulation of this Principle given by C. W. Will [12]. In this formulation[3] the **EEP** states:

1. Weak Equivalence Principle (**WEP**) is valid.

   This means that the trajectory of a freely falling body (one not acted upon by such forces as electromagnetism and too small to be affected by tidal forces) *is independent of its internal structure and composition.*

2. Local Lorentz Invariance (**LLI**) is valid.

   This means that the outcome of any local non-gravitational experiment *is independent of the velocity of the freely-falling reference frame in which it is performed*

3. Local Position Invariance (**LPI**) is valid.

   This means that the outcome of any local non-gravitational experiment *is independent of where and when in the Universe it is performed.*

The only theories of gravity that can embody **EEP** are those that satisfy the postulates of *metric theories of gravity* [12], which are:

1. Spacetime is endowed with a symmetric metric.

2. The trajectories of freely falling bodies are geodesics of that metric.

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[3]In this formulation this Principle can be experimentally tested.

3. In local freely falling reference frames the non-gravitational laws of physics are those written in the language of Special Relativity (**SRT**).

From the **EEP** it follows the *universal pure metric coupling between matter and gravity*. This admits **GR**, of course, and, at most, some of the so–called *scalar–tensor theories* (these, which respect **EEP**) [13–15] without torsion.

So, torsion is *excluded by latest gravitational experiments* (at least in vacuum if we neglect a cosmological background[4]) which have confirmed **EEP** with such very high precision. In consequence, torsion can be treated only as *a hypothetical field* and is not needed in theory of gravity.

In Section II we give a review of the (theoretical) arguments which in our opinion additionally indicate against introducing of torsion into relativistic theory of gravity. We will conclude from these arguments that torsion rather *should not be introduced into theory of gravity* .

In the paper we will confine to the metric–compatible connection $\omega^i{}_k$. We cannot see any reasons in order to consider more general connections, for example, connexion like proposed in [16]. For a metric–compatible connection $\omega^i{}_k$ we have

$$Dg = 0 \tag{1}$$

and

$$\omega^i{}_k =_{LC} \omega^i{}_k + K^i{}_k, \tag{2}$$

where $_{LC}\omega^i{}_k$ means *Levi–Civita connection* and $K^i{}_k$ denotes *contortion*.

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[4]Torsion is excluded at least in vacuum because if we neglect a cosmological background, then the all gravitational experiment were performed in vacuum. In such a case **ECSK** theory can survive since this theory is identical in vacuum with **GR**. But we think that the arguments which we present in Section II testify against **ECSK** theory also.

By ϑ^i we will denote an Lorentzian coreper; Ψ will denote a matter field and D will mean an exterior covariant derivative. For tensor fields the exterior covariant derivative D reduces to an ordinary covariant derivative ∇.

II. ARGUMENTS WHICH INDICATE AGAINST TORSION

The our main argument uses Ockham's razor and the fact: a "wonderful", the most simple and most symmetric Levi-Civita connection is sufficient for the all physical requirements.

The other our arguments are the following:

1. In the paper [17] the authors have showed that the geometry of free–falls and light propagation supplemented by some (very natural axioms) lead us to Riemannian geometry.

2. *Torsion is topologically trivial.* This means that the topology of a real manifold M and topology vector bundles over M (determined by characteristic classes) *do not depend on torsion.* They depend *on curvature only* and are determined by curvature $_{LC}\Omega^i{}_k$ of the Levi-Civita connection $_{LC}\omega^i{}_k$ [18–20]. Roughly speaking, one can continuously deform any metric–compatible connection (or even general linear connection) into Levi-Civita connections *without changing topological invariants and characteristic classes.* So, torsion is not relevant from the topological point of view.

3. *Torsion is not relevant from the dynamical point of view too.* Namely, one can reformulate every metric theory of gravitation with a metric- compatible connection $\omega^i{}_k$ as a "Levi-Civita theory". Torsion is then treated as *a matter field.* An obvious example is given by **ECSK** theory in the so–called "combined formulation" [21].[5] In general, one can prove [23] that any total Lagrangian of the type

$$L_t = L_g(\vartheta^i, \omega^i{}_k) + L_m(\Psi, D\Psi) \tag{3}$$

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[5]In this formulation **ECSK** theory is dynamically fully equivalent to the ordinary **GR** [22].

admits an unique decomposition into a pure geometric part $\tilde{L}_g(\vartheta^i,{}_{LC}\,\omega^i{}_k)$ containing no torsion plus a generalized matter Lagrangian $\tilde{L}_m(\Psi,{}_{LC}\,D\Psi, K^i{}_k,{}_{LC}\,DK^i{}_k)$ which collects the pure matter terms and all the terms involving torsion

$$L_t = L_g + L_m = \tilde{L}_g + \tilde{L}_m. \tag{4}$$

${}_{LC}D$ means an exterior covariant derivative with respect to the Levi-Civita connection ${}_{LC}\omega^i{}_k$. From the Lagrangian

$$L_t = \tilde{L}_g + \tilde{L}_m \tag{5}$$

there follow the *Levi-Civita equations associated with L_t.*

So, torsion *always can be treated as a matter field.* This point of view has been taken e.g. in [24,25] and it is supported by transfromational properties of torsion: torsion transforms like a matter field.

4. *Symmetry of the energy–momentum tensor of matter.* In Special Relativity (**SRT**) a correct energy–momentum tensor for matter (continuous medium, dust, elastic body, solids) *must be symmetric* [26,27]. One can always get such a tensor starting from the *canonical pair* $({}_cT^{ik},{}_c\,S^{ikl} = (-){}_c S^{kil})$ with properties

$$\partial_k {}_c T^{ik} = 0, \quad {}_c T^{ik} - {}_c T^{ki} = \partial_l S^{ikl}, \tag{6}$$

and by use *Belinfante symmetrization procedure* [21,28-30]

$${}_s T^{ik} = {}_c T^{ik} - \frac{1}{2}\partial_j({}_c S^{ikj} - {}_c S^{ijk} + {}_c S^{jki}), \tag{7}$$

$$S^{ijk} = {}_c S^{ijk} - A^{jki} + A^{ikj} = 0. \tag{8}$$

Here

$$A^{ikj} = \frac{1}{2}({}_c S^{ikj} - {}_c S^{ijk} + {}_c S^{jki}). \tag{9}$$

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The obtained new "pair" $(_sT^{ik}, 0)$ is *the most simple and the most symmetric*. Note that the symmetric tensor $_sT^{ik} =_s T^{ki}$ gives complete description of matter because the spin density tensor $_cS^{ijk}$ is *entirely absorbed* into $_sT^{ik}$ during symmetrization procedure.

It is interesting that one can easily generalize the above symmetrization procedure onto a general metric manifold (M_4, g) [9,21] by using Levi-Civita connection associated with the metric g. The generalized symmetrization procedure has the same form as above with replacing $\eta_{ik} \longrightarrow g_{ik}$, $\partial_i \longrightarrow_{LC} \nabla_i$.

So, one can always get in a metric manifold (M_4, g) a symmetric energy–momentum tensor $_sT^{ik} =_s T^{ki}$ for matter (then, of course, corresponding $S^{ikj} = 0$). Observe that the symmetric tensor $_sT^{ik}$, like as in **SRT**, consists of $_cT^{ik}$ and $_cS^{ikl}$.

The symmetric energy–momentum tensor for matter is uniquely determined by the matter equations of motion [31]. This fact is very important for the uniqueness of the gravitational field equations. Moreover, the symetric energy–momentum tensor is covariantly conserved.

L. Rosenfeld has proved [32] that

$$_sT^{ik} = \frac{\delta L_m}{\delta g_{ik}}, \tag{10}$$

where $L_m = L_m(\Psi, _{LC}D\Psi)$ is a covariant Lagrangian density for matter.

The tensor $_sT^{ik}$ given by (10) is the source in the Einstein equations

$$G_{ik} = \chi_s T_{ik}, \tag{11}$$

where $\chi = \frac{8\pi G}{c^4}$.

Note that these equations *geometrize the both canonical quantities $_cT^{ik}$ and $_cS^{ikl} = (-)_cS^{kil}$ in a some equivalent way* because the tensor $_sT^{ik}$ is built from these two canonical tensors.

So, it is the most natural and most simple to postulate that, in general, the correct energy–momentum tensor for matter is the symmetric tensor $_sT^{ik}$. This leads us to a pure metric, torsion-free, theory of gravity which has the field equations of the form

$$\frac{\delta L_g}{\delta g_{ik}} = \frac{\delta L_m}{\delta g_{ik}}. \tag{12}$$

Then, if we take into account the *universality of the Einstein equations* [33–38], we will end up with General Relativity equations (possible with $\Lambda \neq 0$) which will have a sophisticated, symmetric energy-momentum tensor as a source.

5. A gravitational theory with torsion violates **EEP** which has so very good experimental evidence (up to 10^{-14}).

6. In a space-time with torsion a tangent space $T(P)$ cannot be identified with Minkowskian spacetime because there do not exist holonomic coordinates in which $g_{ik}(P) = \eta_{ik}, \quad \Gamma^i_{kl}(P) = 0$. So, a gravitational theory with torsion *is not a covering theory for* **SRT**. We also lose Fermi coordinates [7,39] in Riemann-Cartan space-time.[6]

7. A low-energetic superstrings gravity needn't torsion. It uses only metric g and scalar fields and can always be formulated as Einstein theory (in "Einstein frame") without torsion.

8. A connection having torsion can be determined neither by its own autoparallells (paths) nor by geodesics [7]. So, one cannot determine a connection which has torsion by observation of the test particles (which move along geodesics or autoparallells).

Torsion leads to ambiguities:

1. *Minimal Coupling Principle* (**MCP**) $\neq$ *Minimal Action Principle* (**MAP**) in a space-time with torsion [29].

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[6]Fermi coordinates realize in **GR** a local (freely-falling) inertial frame in which **SRT** is valid.

MCP can be formulated as follows. In a **SRT** field equations obtained from the **SRT** Lagrangian density $L = L(\Psi, \partial_i \Psi)$ we replace $\partial_i \longrightarrow \nabla_i$, $\eta_{ik} \longrightarrow g_{ik}$ and get *covariant field equations* in (M_4, g).

By **MAP** we mean an application of the Minimal Action Principle (Hamiltonian Principle) to the covariant action integral $S = \int\limits_{\Omega} L(\Psi, \nabla_i \Psi) d^4\Omega$, where $L(\Psi, \nabla_i \Psi)$ is a covariant Lagrangian density obtained from the **SRT** Lagrangian density $L(\Psi, \partial_i \Psi)$ by **MCP**.

It is natural to expect that the field equations in (M_4, g) obtained by using **MCP** on **SRT** equations should coincide with the Euler-Lagrange equations obtained from $L(\Psi, \nabla_i \Psi)$ by **MAP**. *This holds in GR but not in the framework of the Riemann-Cartan geometry.* So, we have there an ambiguity in the field equations.[7]

2. In the framework of the **ECSK** theory of gravity we have four energy-momentum tensors for matter: Hilbert, canonical, combined, formal [21]. Which one is correct?

3. Let us consider now normal coordinates **NC(P)** [7,40] which are so very important in **GR** (See, eg., [41-43]). In the framework of the Riemann-Cartan geometry we have two **NC(P)**: normal coordinates for the Levi-Civita part of the Riemann-Cartan connection **NC($_{\mathbf{LC}}\omega$, P)** and normal coordinates for the symmetric part of the full connection **NC($_{\mathbf{s}}\omega$, P)** [44,45]. Which one has a greater physical meaning?

The above ambiguity of the normal coordinates[8] leads us to ambiguities in superenergy and supermomentum tensors [44,45]. Moreover, the obtained expressions are too complicated for practical using. In fact, we lose here a possibility of efective using of the normal coordinates.

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[7]Axial torsion removes this ambiguity. By (M_4, g) we mean here a general metric manifold; not necessarily Riemannian.

[8]Axial torsion removes this ambiguity.

4. In the framework of the Riemann-Cartan geometry [7]

$$R_{(ik)lm} = R_{ik(lm)} = 0, \tag{13}$$

but

$$R_{iklm} \neq R_{lmik}. \tag{14}$$

The last asymmetry leads to an ambiguity in construction of the so–called "Maxwellian superenergy tensor" for the field R_{iklm} [46]. This tensor is uniquely constructed in **GR** owing to the symmetry $R_{iklm} = R_{lmik}$ and it is proportional to the Bel-Robinson tensor [47,48]. In the framework of the Riemann-Cartan geometry the obtained result depends on which antysymmetric pair of the R_{iklm}, first or second, is used in construction.

5. In a Riemann-Cartan space-time we have geodesics and autoparallells (paths). Hamiltonian Principle demands geodesics as trajectories for the test particles. Then, what about physical meaning of the autoparallells? [9]

6. In a space-time with torsion we have in fact two kinds of parallell displacement defined by

$$dv^k = (-)\Gamma^k_{ij}v^j dx^i \tag{15}$$

and

$$dv^k = (-)\Gamma^k_{ij}v^i dx^j. \tag{16}$$

There follow from that two kinds of absolute (and covariant) differentials

$$\nabla_i v^k = \partial_i v^k + \Gamma^k_{il}v^l \tag{17}$$

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[9]Axial torsion removes this problem.

$$\nabla_i v^k = \partial_i v^k + \Gamma_{li}^k v^l. \tag{18}$$

Which one of the two above possibilities could be eventually realized in Nature?

In practice, one must consequently use one of the two above possibilities (or conventions) during any calculations in order to avoid mistakes. For example, in Hehl's papers

$$\nabla_i \sqrt{|g|} = 0 \tag{19}$$

in the Riemann-Cartan space-time (and it is, of course, correct result). But in the paper [49] you can find

$$\nabla_i \sqrt{|g|} \neq 0. \tag{20}$$

The last result is, of course, uncorrect and it is a consequence of mixing of the two above covariant differentiation.

The other source of the computational mistakes connected with torsion is the following: different Authors use definitions of torsion which differ by sign and by factor $1/2$.

III. CONCLUDING REMARKS

As we have seen, the **GR** model of the space-time has very good experimental confirmation. On the other hand, torsion has no experimental evidence (at least in vacuum) and it is not needed in the theory of gravity. Moreover, introducing of torsion into geometric structure of the space-time leads us to many problems (apart from calculational, of course). Some of these problems removes an axial torsion $A_i = \frac{1}{6}\eta_{iabc}Q^{abc}$, $Q^{[abc]} = Q^{abc}$. So, it would be reasonable to confine themselves to the axial torsion only (If one still want to keep on torsion). This is also supported by an important fact that the matter fields (= Dirac's particles) are coupled only to the axial part of torsion in the Riemann-Cartan space-time. However, if we confine to the axial torsion, then (if you remember

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dynamical triviality of torsion and universality of the Einstein equations) we effectively will end up with **GR** + an additional pseudovector field A_i (or with an additional pseudoscalar field φ if the field A_i is potential, i.e., if $A_i = \partial_i \varphi$) [29]. But **GR** with an additional dynamical pseudovector field A_i yields local gravitational physics which may have both location and velocity-dependent effects [13] unobserved up to now. So, we will finish with the conclusion that the geometric model of the space-time given by ordinary **GR** with "wonderful" Levi-Civita connection *seems to be the most satisfactory.* [10]

Acknowledgments

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[10]During Symposium LISA 3 (Golm, July 10-14, 2000) I have discussed with many Participiants of the Symposium about geometric model of the space-time. They all agreed with this conclusion. See also conclusion about correct theory of gravity and space-time structure given recently in [50].

REFERENCES

[1] A.Trautman, "On the Structure of the Einstein–Cartan Equations", *Istituto Nazionale di Alta Matematica, Symposia Mathematica*, **12** (1973) 139.

[2] F.W.Hehl, *Gen. Relativ. Grav.*, **5** (1973) 333; –ibidem **5** (1974) 491.

[3] J.Garecki, *Gen.Relativ. Grav.*, **22** (1990) 111.

[4] F. W. Hehl et al., "Gravitation and the Poincare' Gauge Field Theory with Quadratic Lagrangian" in "General Relativity and Gravitation" Vol.I. Eds. A. Held, **Plenum Publishing Corporation 1980**.

[5] A. Trautman, "Fiber Bundles, Gauge Fields and Gravitation" in "General Relativity and Gravitation" Vol.I. Eds. A. Held, **Plenum Publishing Corporation 1980**.

[6] K. Hayashi and T. Shirafuji, *Prog. Theor. Phys.*, **64** (1980) 866; **64** (1980) 883; **64** (1980) 1435; **64** (1980) 2222; **65** (1981) 525; **66** (1981) 318; **66** (1981) 2258; **73** (1985) 54; **74** (1985) 852.

[7] J.A. Schouten, "Ricci-Calculus", **Springer-Verlag, Berlin 1954**.

[8] E. Bompiani, "Significato del tensore di torsione in une connesione affine", *Boll, Un. Mat. Ital.* **6** (1951) 273.

[9] D. Grensing and G. Grensing, *Phys. Rev.,* **D 28** (1983) 286.

[10] D. Giulini, "Ashtekar Variables in Classical General Relativity" in "Canonical Gravity: From Classical to Quantum", Eds. J. Ehlers and H. Friedrich, **Springer–Verlag 1994**.

[11] A. Ashtekar,"Quantum Mechanics of Geometry", preprint gr-qc/9901023.

[12] Clifford M. Will, "Theory and experiment in gravitational physics", **Cambridge University Press, Cambridge 1981**.

[13] Clifford M. Will, "The Confrontation Between General Relativity and Experiment: A 1998 Update", preprint gr-qc/9811038.

[14] G. Esposito-Farese, "Comparing Solar-System, Binary-Pulsars, and Gravitational-Wave Test of Gravity", preprint gr-qc/9903058.

[15] T. Damour, "Experimental Tests of Relativistic Gravity", preprint gr-qc/9904057.

[16] F.W.Hehl, J.D. McCrea, E.W.Mielke and Y. Neéman, *Found.Phys.*,**19** (1989) 1075.

[17] J. Ehlers, F. A. E. Pirani, A. Schild, "The geometry of free-fall and light propagation" in "Synge Festschrift", **Oxford University Press, Oxford 1972**.

[18] A. Trautman, "Differential Geometry for Physicists", Stony Brook Lectures, **Bibliopolis, Naples 1984**.

[19] S. Kobayashi and K. Nomizu, "Foundations of Differential Geometry", **Interscience Publishers 1963 (Vol.I) and 1969 (Vol.II)**.

[20] K. Maurin, two articles in "Leksykon Matematyczny". Eds. M. Skwarczyński, **Wiedza Powszechna, Warsaw 1993** (in Polish).

[21] F. W. Hehl, *Rep. Math. Phys.*, **9** (1976) 55.

[22] J. Nester, *Phys. Rev.* **D 16** (1977) 239.

[23] M. A. Schweitzer, "Gauge Theory and Gravitation", **PhD Zürich 1980**.

[24] E. Schrödinger, *Proc. R. Irish Acad.*, **51 A** (1947) 163.

[25] M. Ferraris, J. Kijowski, *Gen. Rel. Grav.*, **14** (1982) 37.

[26] C. W. Misner, K. S. Thorne, J. A. Wheeler, "Gravitation", **W. H. Freeman and Company, San Francisco 1973**.

[27] B. F. Schutz, "A First Course in General Relativity", **Cambridge University Press, Cambridge 1985**.

[28] F. J. Belinfante, *Physica*, **6** (1939) 887.

[29] V. de Sabatta and M. Gasperini, "Introduction to Gravitation", **World Scientific, Singapore 1985**.

[30] W. Kopczyński, A. Trautman, "Space-Time and Gravitation", **PWN, Warsaw 1984** (in Polish).

[31] V. D. Fock, "The Theory of Space, Time and Gravitation", **Pergamon Press, London 1969**.

[32] L. Rosenfeld, *Mem. Acad. Roy. Belg.*, **Cl.Sc., fasc. 6** (1940).

[33] G. Magnano et al., *Gen Rel. Grav.*, **19** (1987) 465.

[34] A. Jakubiec, J. Kijowski, *Phys. Rev.,* **D 37** (1988) 1406.

[35] A. Jakubiec, J. Kijowski, *Journ. Math. Phys.,* **30** (1989) 1073.

[36] A. Borowiec et al., *Gen Rel. Grav.*, **26** (1994) 637.

[37] A. Borowiec et al., *Class. Quantum Grav.*, **15** (1998) 43 (gr-qc/9611067).

[38] A. Borowiec et al., "Alternative Lagrangians for Einstein Metrics", preprint gr-qc/9806116 v2.

[39] Wann-Quan Li and Wei-Tou Ni, *J. Math. Phys.*, **20** (1979) 25.

[40] A.Z. Petrov, "New Methods in General Relativity", **Nauka, Moscow 1966** (In Russian). [English edition of Petrov's book: "Einstein Spaces", Pergamon, New York 1969].

[41] L. Brewin, *Class. Quantum Grav.*, **25** (1998) 3085.

[42] A.I. Nesterov, *Class. Quantum Grav.*, **16** (1999) 465.

[43] V. Müller et al., "A Closed Formula for the Riemann Normal Coordinate Expansion", preprint gr-qc/9712092 v2.

[44] J. Garecki, *Acta Phys. Pol.*, **B 9** (1978) 291.

[45] J. Garecki, *Acta Phys. Pol.*, **B 12** (1981) 739.

[46] J. Garecki, *Class. Quantum Grav.*, **2** (1985) 403.

[47] J. Garecki, *Acta Phys. Pol.*, **B 12** (1981) 1017.

[48] J. Garecki, "Some Remarks on the Bel-Robinson Tensor". An amended version of the Essay which received an "honorable mention" in the 1999 Essay Competition of the Gravity Research Foundation. Preprint gr-qc/0003006. Submitted to *Annalen der Physik*.

[49] A. Saa, *Gen Rel. Grav.*, **29** (1997) 205.

[50] N Straumann, "Reflections on Gravity". Preprint astro-ph/0006423.